MENIIT
NEET | IIT-JEE | FOUNDATION
Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 20-07-2021 (EVENING SHIFT) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E} \quad E_0\hat{i}$ and $\vec{B} \quad B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along

(1) $(\ \hat{j})$ (2) $\hat{j}$ (3) $(\ \hat{k})$ (4) $\hat{k}$

Ans. (1)

Sol. Direction of EM wave in given by direction of $\vec{E} \quad \vec{B}$

Unit vector in direction $\vec{E} \quad \vec{B} \quad \dfrac{\vec{E} \quad \vec{B}}{|\vec{E} \quad \vec{B}|}$

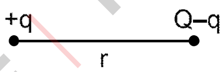
$$\frac{E_0\hat{i} \quad B_0\hat{k}}{E_0B_0 \sin 90} \quad \hat{i} \quad \hat{k} \quad \hat{j}$$

2. The length of a metal wire is l_1, when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is :

(1) $\sqrt{l_1 l_2}$ (2) $\dfrac{l_1 \quad l_2}{2}$ (3) $\dfrac{l_1T_2 \quad l_2T_1}{T_2 \quad T_1}$ (4) $\dfrac{l_1T_2 \quad l_2T_1}{T_2 \quad T_1}$

Ans. (3)

Sol. Let initial length of rod be l_0 and area A.



As a $\dfrac{T}{A} \quad Y\dfrac{\ell}{\ell}$

So, $\dfrac{T_1}{A} \quad \dfrac{Y(l_1 \quad l_0)}{l_0}; \dfrac{T_2}{A} \quad \dfrac{Y(l_2 \quad l_0)}{l_0}$

Dividing $\dfrac{T_1}{T_2} \quad \dfrac{l_1 \quad l_0}{l_2 \quad l_0}; T_1l_2 \quad T_1l_0 \quad T_2l_1 \quad T_2l_0; l_0 \quad \dfrac{l_1T_2 \quad l_2T_1}{T_2 \quad T_1}$

3. With what speed should a galaxy move outward, with respect to earth so that the sodium-D line at wavelength 5890 Å is observed at 5896 Å ?

(1) 322 km/sec (2) 296 km/sec (3) 306 km/sec (4) 336 km/sec

Ans. (3)

Sol. $\dfrac{V_{rol}}{C} \quad \dfrac{}{}$

$$V_{rol} \quad \frac{6}{5890} \quad 3 \quad 10^8 \quad 306\text{km/s}$$

4. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be

(1) 400% (2) 100% (3) 300% (4) 200%

Ans. (2)

Sol. K.E. $K \quad \frac{P^2}{2m}$

$P \quad \sqrt{K}$

$\frac{P_2}{P_1} \quad \sqrt{\frac{K_2}{K_1}} \quad \frac{P_2}{P_1} \quad \sqrt{\frac{4K}{K}}$

$\frac{P_2}{P_1} \quad 2$

$\frac{P_2 \quad P_1}{P_1}\% \quad \frac{P_2}{P_1} \quad 1 \quad 100 \quad (2 \quad 1) \quad 100 \quad 100$

$\frac{P}{P_1}\% \quad 100\%$

5. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} \quad \vec{Q}$ is n times the magnitude of $\vec{P} \quad \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :

(1) $\cos^{1} \frac{n \quad 1}{n \quad 1}$ (2) $\sin^{1} \frac{n \quad 1}{n \quad 1}$ (3) $\cos^{1} \frac{n^2 \quad 1}{n^2 \quad 1}$ (4) $\sin^{1} \frac{n^2 \quad 1}{n^2 \quad 1}$

Ans. (3)

Sol. $|\vec{P} \quad \vec{Q}| \quad n|\vec{P} \quad \vec{Q}|$

$|\vec{P}|^2 \quad |\vec{Q}|^2 \quad 2|\vec{P}||\vec{Q}|\cos \quad n^2 \quad |\vec{P}|^2 \quad |\vec{Q}|^2 \quad 2|\vec{P}||\vec{Q}|\cos$

$2 + 2\cos\theta = n^2 (2 - 2\cos\theta)$

$\cos^{1} \frac{n^2 \quad 1}{n^2 \quad 1}$

6. At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip :

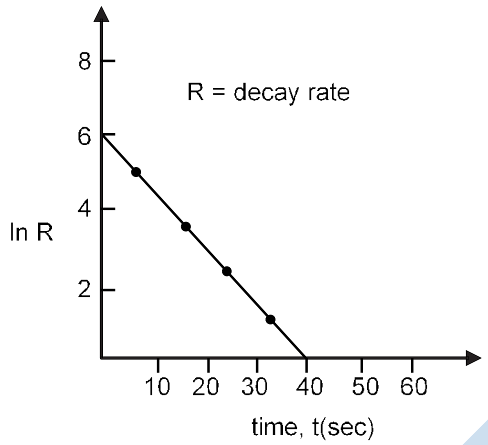
(1) $\tan^{1} \frac{\sqrt{3}}{2}$ (2) $\tan^{1} \frac{1}{\sqrt{3}}$ (3) $\tan^{1} \sqrt{3}$ (4) $\tan^{1} \frac{2}{\sqrt{3}}$

Ans. (1)

Sol. $\tan 45^\circ \quad \frac{B\sin}{Bcoo \quad \cos 30^\circ}$

$\tan \quad \frac{\sqrt{3}}{2}$

7. For a certain radioactive process the graph between ln R and t(sec) is obtained as shown in the figure. Then the value of half-life for the unknown radioactive material is approximately :



(1) 4.62 sec (2) 2.62 sec (3) 9.15 sec (4) 6.93 sec

Ans. (1)

Sol. $R = Roe^{-\lambda t}$

$\ln R = \ln R_0 - \lambda t$

slope $\frac{6}{40}; \frac{3}{20}$

$t_{1/2} \quad \frac{\ln 2}{} \quad \frac{\ln 2}{3} \quad 20 \quad \frac{20}{3}\ln 2 \quad 4.62\,\text{sec}$

8. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod is :

(1) $\pi \times 10^{-4}$ H/m (2) $3\pi \times 10^{-4}$ H/m (3) $4\pi \times 10^{-4}$ H/m (4) $2\pi \times 10^{-4}$ H/m

Ans. (4)

Sol. $\mu_r = 1 + \chi$

$= 1 + 499 = 500$

Absolute permeability $= \mu_0 + \mu_r$

$500 \times 4\pi \times 10^{-7} = 2\pi \times 10^{-4}$ H/m

9. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to :

(1) $t^{\frac{1}{2}}$ (2) $t^{\frac{1}{4}}$ (3) $t^{\frac{3}{2}}$ (4) $t^{\frac{3}{4}}$

Ans. (3)

Sol. Energy supply = Pt

In t sec

$Pt \quad \frac{1}{2}mv^2; V \quad \sqrt{t}; \frac{dS}{dt} \quad C\sqrt{t}$

$$\int_0^S dS \quad C \int_0^t t^{1/2}dt \quad S \quad \frac{2Ct^{3/2}}{3}$$

$S \propto t^{3/2}$

10. The correct relation between the degrees of freedom f and the ratio of specific heat γ is :

(1) $f \quad \frac{1}{2}$ (2) $f \quad \frac{1}{1}$ (3) $f \quad \frac{2}{1}$ (4) $f \quad \frac{2}{1}$

Ans. (4)

Sol. $Cv \quad \frac{fR}{2}$

$C_P \quad \frac{f}{2} \quad 1 \; R$

$\frac{C_P}{C_v} \quad 1 \quad \frac{2}{f}$

$1 \quad \frac{2}{f}$

$f \quad \frac{2}{1}$

11. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then ;

(1) $T_A = T_B$ (2) $T_A > T_B$ (if $r_A > r_B$) (3) $\frac{T_A}{T_B} \quad \frac{r_A}{r_B}^{\frac{3}{2}}$ (4) $T_A > T_B$ (if $m_A > m_B$)

Ans. (1)

Sol. Both the planets will move due to mutual gravitational interaction, thus will have same angular velocity. This will result in same time period.

12. An electron having de-Broglie wavelength λ is incident on a target in a X-ray tube. Cut-of wavelength of emitted X-ray is :

(1) $\frac{2m^2c^2 \quad ^2}{h^2}$ (2) $\frac{hc}{mc}$ (3) 0 (4) $\frac{2mc \quad ^2}{h}$

Ans. (4)

Sol. De-broglie wavelength

$\frac{h}{P} \quad P \quad \frac{h}{}$

$\therefore$ Kinetic energy of electron $\quad E \quad \frac{P^2}{2m} \quad \frac{h^2}{2m \quad ^2}$

For cut-off wavelength of emitted X-Ray

$E \quad \frac{hc}{}$

$\frac{h^2}{2m\ \ ^2}$ $\quad \frac{hc}{}$ $\quad \frac{2mc\ \ ^2}{h}$

13. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

(1) $t_2 - t_1$ (2) $\frac{t_1t_2}{t_2 \quad t_1}$ (3) $\frac{t_1t_2}{t_2 \quad t_1}$ (4) $\frac{t_1 \quad t_2}{2}$

Ans. (3)

Sol. Suppose length of escalator = L

Speed of man w.r.t escalator $\frac{L}{t_1}$

Speed of escalator $\frac{L}{t_2}$

Speed of man w.r.t. ground when escalator is moving $\frac{L}{t_1} \quad \frac{L}{t_2}$

Time taken by the man to walk on the moving escalator $\frac{L}{\frac{L}{t_1} \quad \frac{L}{t_2}} \quad \frac{t_1t_2}{t_1 \quad t_2}$

14. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is

(1) 1.5 (2) 0.7 (3) 3.0 (4) 2.0

Ans. (3)

Sol. As $T \propto R^{3/2}$

$T_1 = kR^{3/2}$

$\frac{T}{T} \quad \frac{3}{2} \quad \frac{R}{R} \quad 3\%$

15. Two small drops of mercury each of radius R coalesce to form a single large drop. The ratio of total surface energy before and after the change is :

(1) $1:2^{\frac{1}{3}}$ (2) $2^{\frac{1}{3}}:1$ (3) 2 : 1 (4) 1 : 2

Ans. (2)

Sol. $2 \quad \frac{4}{3} \quad R^3 \quad \frac{4}{3} \quad r^3$

$\frac{R}{r} \quad \left(\frac{1}{2}\right)^{\frac{1}{3}}$ .....(1)

Now $\frac{U_{before}}{U_{after}} \quad \frac{2 \quad S \quad 4 \quad R^2}{S \quad 4 \quad r^2} \quad 2 \quad \left(\frac{R}{r}\right)^2 \quad \frac{2^{1/3}}{1}$

16. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

(1) Hollow cylinder (2) Solid sphere (3) Sold cylinder (4) Ring

Ans. (3)

Sol. Given $\frac{1}{2}I\ {}^{2}\quad \frac{1}{2}\ \frac{1}{2}mv^2$

as v = Rω (pure rolling)

$\frac{1}{2}I\ {}^{2}\quad \frac{1}{4}mR^2\ {}^{2};I\quad \frac{1}{2}mR^2$

Thus, solid cylinder.

17. For a series LCR circuit with R = 100 Ω, L = 0.5 mH ad C = 0.1 pF connected across 220 V – 50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is :

(1) 0°, resonance circuit

(2) ≈ 90°, predominantly capacitive circuit

(3) 0°, resistive circuit

(4) ≈ 90°, predominantly inductive circuit

Ans. (2)

Sol. $\omega = 2\pi f = 100\ \pi$

$R\quad 100\ ,X_L\quad L\quad \frac{100\quad 0.5}{1000}\quad \frac{}{200},X_C\quad \frac{1}{C}\quad \frac{10^{12}}{100\quad 0.1}\quad \frac{10^{11}}{}$

As $X_C >> X_L$ & $X_C >> R$

Then tan $\frac{|X_L\quad X_C|}{R}$; 90°

18. If time (t), velocity (v), and angular momentum (ℓ) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v, and l is :

(1) $[t^{-1}\ v^{-2}\ \ell^{1}]$ (2) $[t^{-2}\ v^{-1}\ \ell^{1}]$ (3) $[t^{-1}\ v^{2}\ \ell^{-1}]$ (4) $[t^{-1}\ v^{1}\ \ell^{-2}]$

Ans. (1)

Sol. $M \propto t^x\ v^y\ \ell^z$

$M^0L^0T^0 = t^x\ [LT^{-1}]^y\ [ML^2T^{-1}]^z$

$M^1L^0T^0 = t^{x-y-z}\ L^{y+2z}\ M^z$

On comparing powers

$z = 1$ ...(1)

$x - y - z = 0$ ...(2)

$y + 2z = 0$ ...(3)

$y + 2 \times 1 = 0$

$y = -2$

$x - (-2) - 1 = 0$

$x = -1$; $M \propto t^{-1}\ v^{-2}\ \ell^{1}$; $[M] \propto [t^{-1}\ v^{-2}\ \ell]$

19. A particle is making simple harmonic motion along, the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are υ_1 and υ_2 respectively. The time period of its oscillation is given as :

(1) $T \quad 2 \sqrt{\frac{x_2^2 \quad x_1^2}{\ _1^2 \quad \ _2^2}}$ (2) $T \quad 2 \sqrt{\frac{x_2^2 \quad x_1^2}{\ _1^2 \quad \ _2^2}}$ (3) $T \quad 2 \sqrt{\frac{x_2^2 \quad x_1^2}{\ _1^2 \quad \ _2^2}}$ (4) $T \quad 2 \sqrt{\frac{x_2^2 \quad x_1^2}{\ _1^2 \quad \ _2^2}}$

Ans. (4)

Sol. $v \quad \sqrt{A^2 \quad x^2}$

$v_1 \quad \sqrt{A^2 \quad x_1^2}$

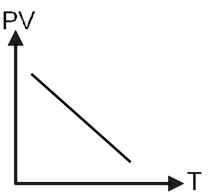
$v_2 \quad \sqrt{A^2 \quad x_2^2}$

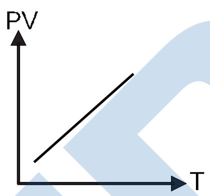
$\frac{v_1}{\ }^2 \quad \frac{v_2}{\ }^2 \quad x_2^2 \quad x_1^2$

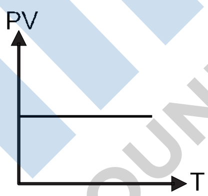
$^2 \quad \frac{v_1^2 \quad v_2^2}{x_2^2 \quad x_1^2}$

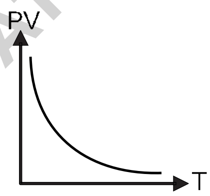
$\sqrt{\frac{v_1^2 \quad v_2^2}{x_2^2 \quad x_1^2}}; T \quad 2 \sqrt{\frac{x_2^2 \quad x_1^2}{\ _1^2 \quad \ _2^2}}$

20. Which of the following graphs represent the behaviour of an ideal gas ? Symbols have their usual meaning.

(1)


(2)


(3)


(4)


Ans. (2)

Sol. PV = nRT

$\Rightarrow$ PV = CT

Therefore, PV v/s T graph is straight line.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping and inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is ___________ .

Ans. (3)

Sol. $a \quad \frac{qE}{m} \quad \frac{8 \quad 10^{6}}{10^{3}} \quad 100 \quad 0.8\,m/s^{2}$

As electric field is switched on, ball first strikes to wall and returns back.

On oscillation.

Thus $s \quad ut_1 \quad \frac{1}{2}at_1^2$

$0.1 \quad \frac{1}{2} \quad 0.8t_1^2; t_1 \quad \frac{1}{2}s$

Thus time period $T \quad 2 \quad \frac{1}{2} \quad 1\text{sec}$.

2. A body of mass 'm' is launched up on a rough inclined plane making and angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is _________ .

Ans. (3)

Sol. $S \quad \frac{1}{2}a_A t_A^2$ ....(1)

$S \quad \frac{1}{2}a_D t_D^2$ ....(2)

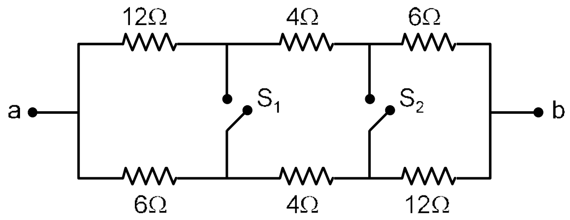
From equation (1) & (2)

$\frac{t_A^2}{t_D^2} \quad \frac{a_D}{a_A}$

$\Rightarrow \quad \frac{t_A^2}{t_D^2} \quad \frac{g\sin \quad \mu g\cos}{g\sin \quad \mu g\cos} \quad \frac{t_A}{t_D} \quad \sqrt{\frac{g\sin \quad \mu g\cos}{g\sin \quad \mu g\cos}}$

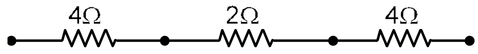
$\Rightarrow \quad \frac{1}{2} \quad \sqrt{\frac{1 \quad \sqrt{3}\mu}{1 \quad \sqrt{3}\mu}} \quad 1 \quad \sqrt{3}\mu \quad 4 \quad 4\sqrt{3}\mu \quad 5\sqrt{3}\mu \quad 3 \quad \mu \quad \frac{\sqrt{3}}{5}$

3. In the given figure switches S_1 and S_2 are in open condition. The resistance across ab when the switches S_1 and S_2 are closed is __________ Ω.



Ans. (10)

Sol.



= 10 Ω

4. A series LCR circuit of R = 5 Ω, L = 20 mH and C = 0.5 μF is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is __________ × 10^2 W.

Ans. 125

Sol. As circuit is in resonance. Thus

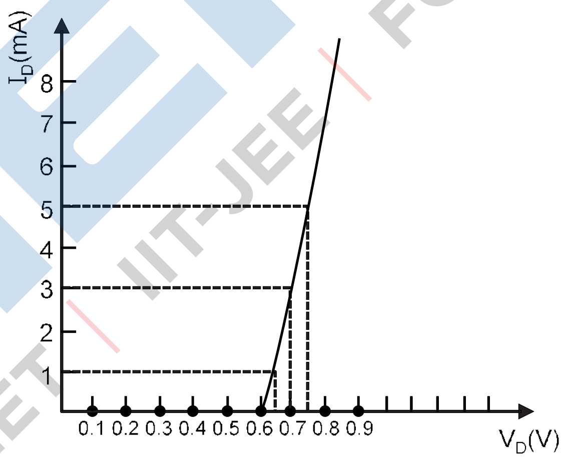
$X_L = X_C$

∴ Z = R so i_{rms} = V/Z = V/R

$P \quad i_{rms}^2 R$

$P \quad \frac{V^2}{R} \quad \frac{250 \quad 250}{5} \quad 12500 J/s \quad 125 \quad 10^2 W$

5. For the forward biased diode characteristics shown in the figure, the dynamic resistance at I_D = 3 mÅ will be __________ Ω.



Ans. (25)

Sol. Dynamic resistance $\frac{V}{I}$

$\frac{0.75 \quad 0.65}{(5 \quad 1) \quad 10^{3}} \quad 25$

6. A certain metallic surface is illuminated by monochromatic radiation of wavelength λ. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength 2λ, the stopping potential is V_0, The threshold wavelength of this surface for photoelectric effect is ______ λ.

Ans. (4)

Sol. KE = hν – W

$eV \quad \frac{hc}{} \quad W$

For first case ; $e(3V_0) \quad \frac{hc}{} \quad W$ ....(i)

For Second case : $eV_0 \quad \frac{hc}{2} \quad W$ ....(ii)

From equation (i) and (ii)

$W \quad \frac{hc}{4}$

For λ_{th} ; $W \quad \frac{hc}{_{th}}$

$\frac{hc}{4} \quad \frac{hc}{_{th}} \quad _{th} \quad 4$

7. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is ________ .

Ans. (200)

Sol. ω_0 = 600 rpm = 10 rps

ω = 1800 rpm = 30 rps

$\frac{30 \quad 10}{10} \quad 2rps^2$

$_0t \quad \frac{1}{2} \quad t^2 \quad 10 \quad 10 \quad \frac{1}{2} \quad 2 \quad (100) \quad 200$

8. A radioactive substance decays to $\frac{1}{16}^{th}$ of its initial activity in 80 days. The half life of the radioactive substance expressed in days is _______ .

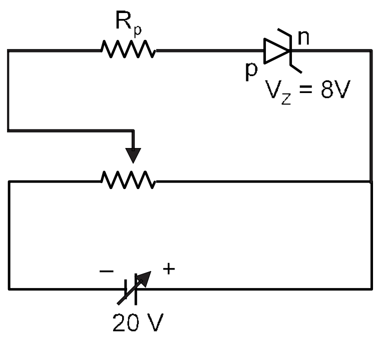
Ans. (20)

Sol. $N_0 \quad t_{1/2} \quad \frac{N_0}{2} \quad t_{1/2} \quad \frac{N}{4} \quad t_{1/2} \quad \frac{N}{8} \quad t_{1/2} \quad \frac{N}{16}$

$4 \times t_{1/2}$ = 80 days

$t_{1/2}$ = 20 days

9. A zener diode having zener voltage 8 v and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance R_P is _____________ Ω.



Ans. (192)

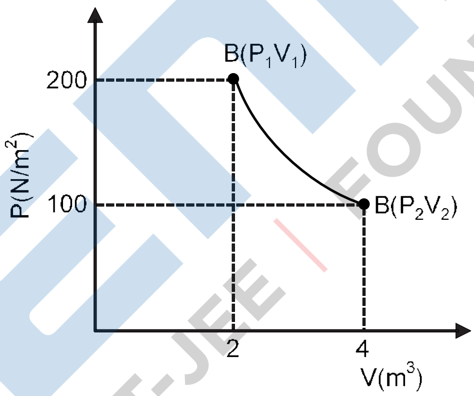
Sol. $i \quad \frac{20}{128 \quad R_p}$

$12 \quad \frac{20}{128 \quad R_p} \quad R_p$

$1536 = 8R_p$

$R_p = 192$

10. One mole of an ideal gas at 27°C is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be __________ $\times 10^{-1}$ J. [Given : R = 8.3 J/mole K, ln2 = 0.6931]



Ans. (1725.8)

Sol. $W \quad nRT\ell n\frac{V_2}{V_1}$

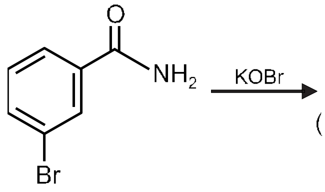
$= 1 \times 8.3 \times 300 \ln 2$

$= 1725.8$ J

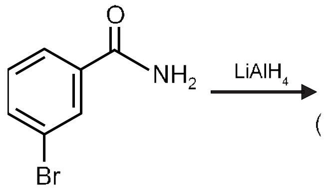
PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

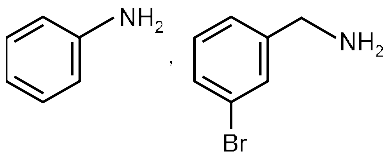
1. 

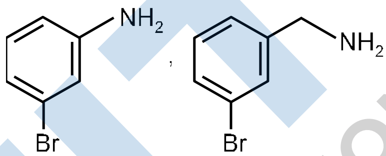
A (major product)

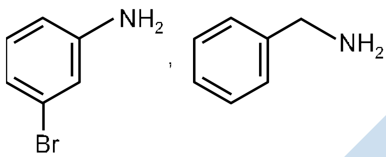


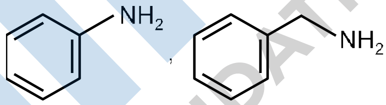
B (major product)

In the above reactions, product A and product B respectively are :

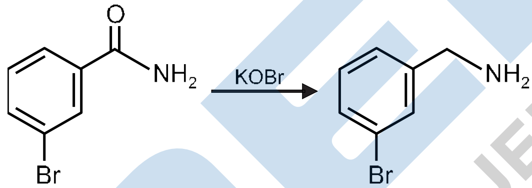
(1) 

(2) 

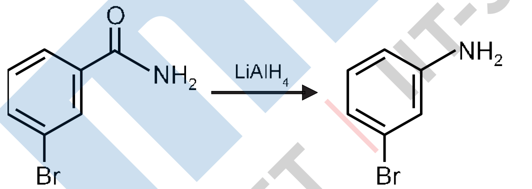
(3) 

(4) 

Ans. (2)

Sol. 

(Hofmann bromamic



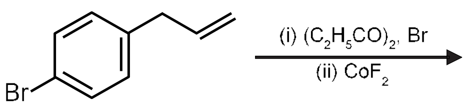
(Reduction) – LiAlH,

2. Metallic sodium does not react normally with :

(1) Tert-butyl alcohol (2) Gaseous ammonia (3) But-2-yne (4) Ethyne

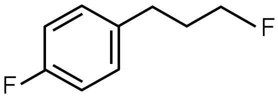
Ans. (3)

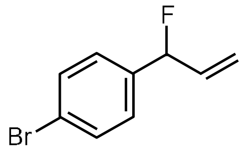
Sol. But-2-yne does not have any acidic hydrogen.

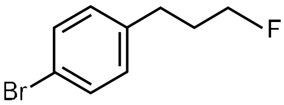
3. 

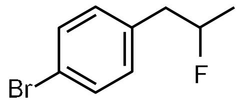
P (major product)

Major product P of above reaction, is :

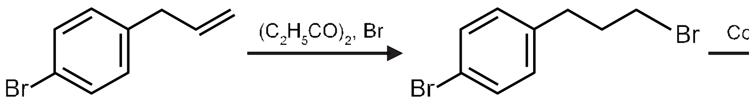
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

4. Outermost electronic configuration of a group 13 element, E, is $4s^2$, $4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is :

(1) [Xe] $5d^{10}$ $6s^2$ $6p^2$ (2) [Ar] $3d^{10}$ $4s^2$ $4p^2$ (3) [Kr] $3d^{10}$ $4s^2$ $4p^2$ (4) [Kr] $4d^{10}$ $5s^2$ $5p^2$

Ans. (4)

Sol.

		13th	14th	15th
2nd period	$2s^22p^1$	B	C	N
3rd period	$3s^23p^1$	Al	Si	P
4th period	$4s^24p^1$	Ga ↘	Ge	As
5th period	$5s^25p^1$	In	Sn	Sb

That element is ${}_{50}Sn \Rightarrow$ [Kr] $4d^{10}$ $5s^2$ $5p^2$

5. Consider two chemical reactions (A) and (B) that take place during metallurgical process :

(A) $ZnCO_3(g)$ $ZnO(s) + CO_2(g)$

(B) $2ZnS(s) + 3O_2(g)$ $2ZnO(s) + 2SO_2(g)$

The correct option of names give to them respectively is :

(1) Both (A) and (B) are producing same product so both are calcination

(2) Both (A) and (B) are producing same product so both are roasting

(3) (A) is roasting and (B) is calcination

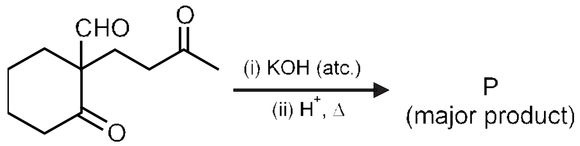
(4) (A) is calcination and (B) is roasting

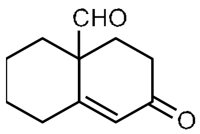
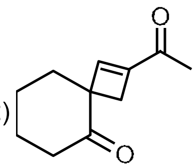
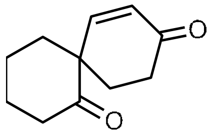
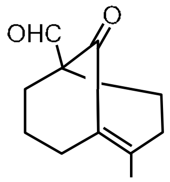
Ans. (4)

Sol. (A) Calcination $\Rightarrow ZnCO_3(g)$ $ZnO(s) + CO_2(g)$

(B) Roasting $\Rightarrow 2ZnS(s) + 3O_2(g)$ $2ZnO(s) + 2SO_2(g)$

6. The major product (P) in the following reaction is :

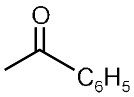
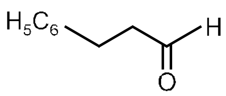


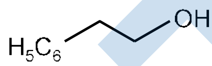
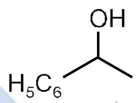
(1)  (2)  (3)  (4) 

Ans. (3)

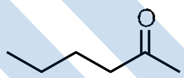
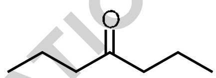
Sol. Intramolecular aldol condensation.

7. Which one of the following pairs of isomers is an example of metamerism ?

(1)  and 

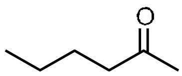
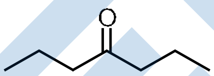
(2)  and 

(3) $CH_3CH_2CH_2CH_2CH_3$ and $H_3C-\underset{CH_3}{\overset{CH_3}{C}}-CH_3$

(4)  and 

Ans. (4)

Sol. Metamers are compounds which have different alkyl groups present along both side of polyvalent functional group.

 and  – Metamers

8. The single largest industrial application of dihydrogen is :

(1) Rocket fuel in space research
(2) Manufacture of metal hydrides
(3) In the synthesis of ammonia
(4) In the synthesis of nitric acid

Ans. (3)

Sol. The largest single industrial application of dihydrogen in the synthesis of ammonia [NCERT page 287]

9. Which one of the following statement is not true about enzymes ?

(1) Enzymes are non-specific for a reaction and substrate.
(2) Almost all enzymes are proteins
(3) The action of enzymes is temperature and pH specific
(4) Enzymes work as catalysis by lowering the activation energy of a biochemical reaction.

Ans. (1)

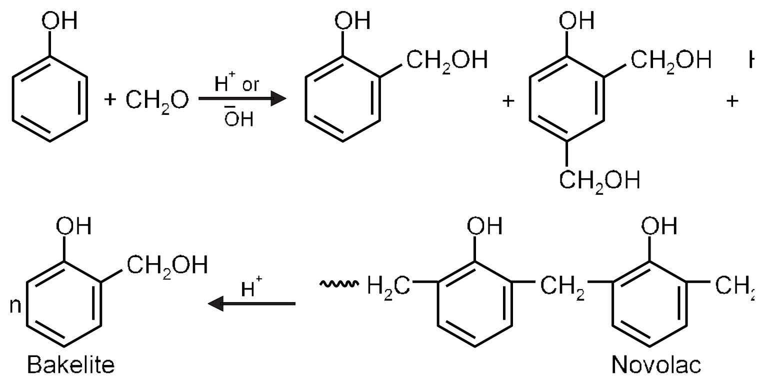
Sol. Enzymes are highly specific in nature

10. Bakelite is a cross-linked polymer of formaldehyde and :

(1) PHBV (2) Dacron (3) Buna-S (4) Novolac

Ans. (4)

Sol.



11. Benzene on nitration gives nitrobenzene in presence of HNO_3 and H_2SO_4 mixture, where :

(1) both H_2SO_4 and HNO_3 act as a bases (2) HNO_3 acts as a base and H_2SO_4 acts as an acid

(3) both H_2SO_4 and HNO_3 act as a acids (4) HNO_3 acts as a acid and H_2SO_4 acts as a base

Ans. (2)

Sol. $HNO_3 + H_2SO_4 \longrightarrow NO_2^+ + HSO_4^- + H_2O$

12. Spin only magnetic moment of an octahedral complex of Fe2+ in the presence of a strong filed ligand in BM is :

(1) 0 (2) 2.82 (3) 4.89 (4) 3.46

Ans. (1)

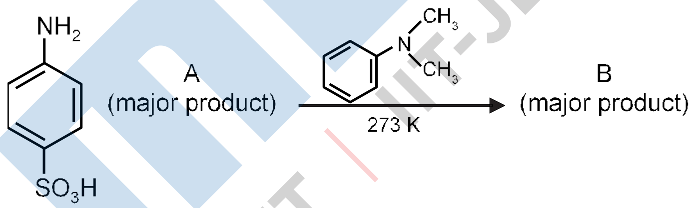
Sol. Fe^{2+} = |Ar|, $3d^6$

in presence of strong filed ligand $Fe^{2+} = t_{2g}^{2,2,2}, e_g^{0,0}$

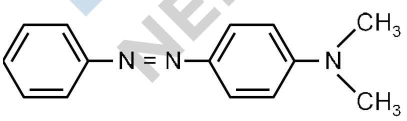
Unpaired e^- [n = 0]

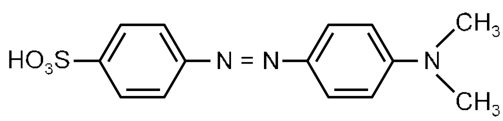
$\mu \quad \sqrt{n(n \quad 2)} \quad 0$

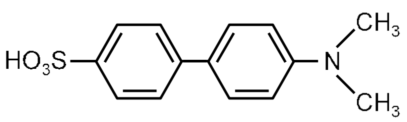
13.



Consider the above reaction, compound B is :

(1)


(2)


(3)


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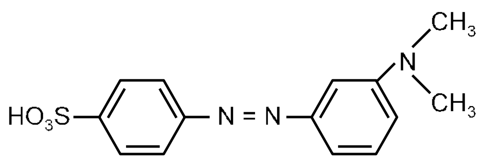
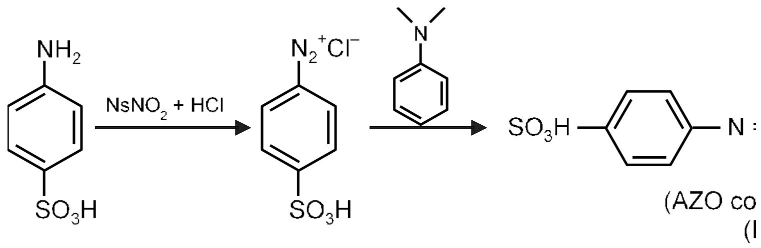


Ans. (2)

Sol.



Diazotisation reaction

14. Which one of the following gases is reported to retard photosynethesis ?

(1) CO_2 (2) CO (3) CFCs (4) NO_2

Ans. (4)

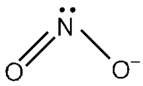
Sol. **Reason :** NO_2 damage the leaves of plants and retard the photosynthesis.

15. The hybridisation of the atomic orbitals of nitrogen in NO_2^-, NO_2^+ and NH_4^+ respectively are :

(1) sp^2, sp and sp^3 (2) sp, sp^2 and sp^3 (3) sp^3, sp and sp^2 (4) sp^3, sp^2 and sp

Ans. (1)

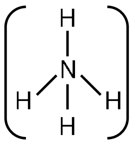
Sol. NO_2^-



sp^2

NO_2^+ $O = \overset{\oplus}{N} = O$ sp

NO_2^+



sp^3

16. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :

(1) $BaSO_4$ (2) HNO_3 (3) $AgNO_3$ (4) $CuSO_4$

Ans. (3)

17. Cu^{2+} salt reacts with potassium iodide to give :

(1) CuI (2) Cu_2I_2 (3) $Cu(I_3)_2$ (4) Cu_2I_3

Ans. (2)

Sol. $2Cu^{2+} + 4KI \longrightarrow Cu_2I_2(S) + I_2 + 4K^+$

18. A solution is 0.1 M in Cl^- and 0.001 M in CrO_4^{2-}. Solid $AgNO_3$ is gradually added to it. Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10}$ M^2 and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12}$ M^3. Select correct statement from the following :

(1) AgCl will precipitate first as the amount of Ag^+ needed to precipitate is low.

(2) AgCl precipitate first because its K_{sp} is high.

(3) Ag_2CrO_4 precipitates first because the amount of Ag^+ needed is low.

(4) Ag_2CrO_4 precipitate first as its K_{sp} is low.

Ans. (1)

Sol. (i) Concentration of Ag^+ required for precipitation of AgCl

$K_{sp}(AgCl) = [Ag^+] [Cl^-]$

$1.7 \times 10^{-10} = [Ag^+] (0.1)$

$[Ag^+] = 1.7 \times 10^{-9}$ M

(ii) Concentration of Ag+ required for precipitation of Ag_2CrO_4

$K_{sp}(Ag_2CrO_4) = [Ag^+]2\ [CrO_4^{2-}]$

$1.9 \times 10^{-12} = [Ag^+]2\ (0.001)$

$[Ag\ ]\ \ \sqrt{1.9\ \ 10^{\ 9}}\ \ \ \sqrt{19}\ \ 10^{\ 5}$

So, AgCl get precipitated first.

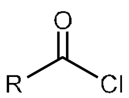
19. Which one of the following species doesn't have a magnetic moment of 1.73 BM, (spin only value) ?

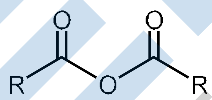
(1) O_2^- (2) CuI (3) O_2^+ (4) $[Cu(NH_3)_4]Cl_2$

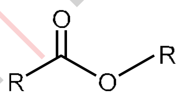
Ans. (2)

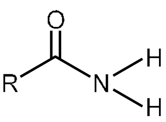
Sol. μ = 1.73 BM It means number of unpaired electron = 1

Species	**unpaired electron**
O_2^-	1
O_2^+	1
Cu^+	0
Cu^{2+}	1

20. (A) 

(B) 

(C) 

(D) R 

The correct order of their reactivity towards hydrolysis at room temperature is :

(1) (D) > (A) > (B) > (C) (2) (A) > (C) > (B) > (D)

(3) (D) > (B) > (A) > (C) (4) (A) > (B) > (C) > (D)

Ans. (4)

Sol. Rate of hydrolysis is directly proportional to δ positive charged present on carbon of C = O group. Rate of hydrolysis – Acid chloride > Acid anhydride > ester > amide

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The vapour pressures of A and B at 25ºC are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is $x \times 10^{-1}$. The value of x is __________ .

Ans. (1)

Sol. $P^0{}_A = 90$ & $P^0{}_B = 15$

$P_{total} = 90 \times 0.6 + (15)\ 0.4$

$= 54 + 6 = 60$ mm of Hg

$P_B = P_{total}\ Y_B = P^0{}_B Y_B$

$Y_B \quad \frac{15 \quad 0.4}{60} \quad 0.1$

$= 0.1$

$= 1 \times 10^{-1}$

2. 4 g equimolar mixture of NaOH and Na_2CO_3 contains x g of NaOH and y g of Na_2CO_3. The value of x is ________ g.

Ans. (1)

Sol. Given (i) $x + y = 4$

(ii) $\frac{X}{40} \quad \frac{Y}{106}$ [Equimolar]

$Y \quad \frac{106}{40} \ X$

So $X \quad \frac{106}{40} X \quad 4$

$X = 2.065\ X = 4$

$3.65\ X = 4$

$X = 1.096$ gram.

3. For a given chemical reaction $A \rightleftharpoons B$ at 300 K the free energy change is -49.4 kJ mol^{-1} and the enthalpy of reaction is 51.4 kJ mol^{-1}. The entropy change of the reaction is ________ JK^{-1} mol^{-1}.

Ans. (336)

Sol. $\Delta G^o = \Delta H^o - T\Delta S^o$

$-49.4 = 51.4 - T\Delta S^o$

$S^o \quad \frac{49.4 \quad 51.4}{300}$

$= 0.336$ KJ/K = 336 J/K

4. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is ________.

Ans. (8)

Sol. In diamond carbon formed : FCC unit cell + 50% (TV) are occupied

effective number of atoms of carbon = $4 \quad 8 \quad \frac{1}{2} \quad 8$

5. The wavelengths of electrons accelerated from rest through a potential difference of 40 kV is x × 10^{-12} m. The value of x is ________.

Given : Mass of electron = 9.1 × 10^{-31} kg

Charge on an electron = 1.6 × 10^{-19} kg

Planck's constant = 6.63 × 10^{-34} Js

Ans. (6)

Sol. $\frac{12.3}{\sqrt{v}}$

$v = 40 \times 10^3$

$$\frac{12.3}{\sqrt{40 \quad 10^3}} \quad \frac{12.3}{2 \quad 10^2}$$

$= 0.0615$ Å $= 6.15 \times 10^{-12}$ m

6. 100 ml 0.0018% (w/v) solution of Cl^- ion was the minimum concentration of Cl^- required to precipitate a negative sol in one h. The coagulating value of Cl^- ion is ________.

Ans. (1)

Sol. **Coagulation value :** The minimum concentration of electrolyte in milimoles required to cause coagulation of 1 lit. of colloidal solution

Given : 0.0018 gram Cl^- present in 100 ml solution.

$$\text{Coagulation value of Cl} \quad \frac{\frac{0.0018}{35.5} \quad 10^3}{0.1} \quad 0.5070$$

7. PCl5(g) $\longrightarrow$ $PCl_3(g) + Cl_2(g)$

In the above first order reaction the concentration of PCl_5 reduces from initial concentration 50 mol L^{-1} to 10 mol L^{-1} in 120 minutes at 300 K. The rate constant for the reaction at 300 K is x × 10^{-2} min^{-1}. The value of x is ________. [Given log5 = 0.6989]

Ans. (1)

Sol. $PCl_5(g) \longrightarrow PCl_3(g) + Cl_2(g)$

t = 0 — 50 moles

t = 120 minutes — 10 mole

$$K \quad \frac{1}{t}\ln\frac{a}{a \quad x} \qquad \frac{2.303}{120}\log\frac{50}{10}$$

$$\frac{2.303 \quad 0.693}{120} \quad 0.0133 \text{ minutes}^{-1}$$

$= 1.33 \times 10^{-2}$ minutes^{-1} $\approx$ 1 minutes^{-1}

8. When 0.15 g of an organic compound was analysed using Carius method estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is ______.

(Atomic mass : Silver = 108, Bromine = 80)

Ans. (68)

Sol. $\underset{0.15 \text{ gram}}{\text{Organic compound}} \xrightarrow{AgNO_3} \underset{0.2397 \text{ gram}}{AgBr}$

Mole of AgBr $\quad \dfrac{0.2397}{188}$

Mole of Br $\quad \dfrac{0.2397}{188} \quad 80 \quad 0.102$gram

% of Bromine in organic $\quad \dfrac{0.102}{0.15} \quad 100 \quad 68$

9. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.

$$6OH^- + Cl^- \longrightarrow ClO_3^- + 3H_2O + 6e^-$$

A current of xA has to be passed for 10 h to produce 10.0 g of potassium chlorate. The value of x is _______.

(Molar mass of $KClO_3$ = 122.6 g mol^{-1}, F = 96500 C)

Ans. (1)

Sol. $6OH^- + Cl^- \longrightarrow ClO_3^- + 3H_2O + 6e^-$

$w \quad \dfrac{E}{96500} \quad i \quad t$

$10 \quad \dfrac{122.6}{6 \quad 96500} \quad x \quad 10 \quad 60 \quad 60$

x = 1.31 A

10. An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $[Ni(CN)_6]^{2-}$. The total change in number of unpaired electron on metal centre is _______.

Ans. (2)

Sol. $NiCl_2 \quad Ni^{2} \quad 3d^8$ [| | | |]

Unpaired electron $\boxed{n \quad 2}$

$[Ni(CN)_6]^{2-} \Rightarrow Ni^{4+} \Rightarrow 3d^6 \Rightarrow t_{2g}^{2,2,2}, eg^{0,0}$

unpaired electron = 0

difference in unpaired electron = 2

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Let y = y(x) satisfies the equation $\frac{dy}{dx} + |A| = 0$, for all x > 0, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If y(π) = π + 2, then the value of $y\left(\frac{\pi}{2}\right)$ is :

(1) $\frac{\pi}{2} - \frac{4}{\pi}$ (2) $\frac{3\pi}{2} - \frac{1}{\pi}$ (3) $\frac{\pi}{2} - \frac{1}{\pi}$ (4) $\frac{\pi}{2} + \frac{4}{\pi}$

Ans. (4)

Sol. $|A| = y\left(-\frac{1}{x}\right) - \sin x(-2) + 1(2)$

$|A| = -\frac{y}{x} + 2\sin x + 2$

$\frac{dy}{dx} - \frac{t}{x} + 2\sin x + 2 = 0$

$\frac{dy}{dx} - \frac{y}{x} + 2(\sin x + 1) = 0$

IF. $= e^{\int \frac{1}{x}dx} = x$

$yx = \int -2(\sin x + 1)x\,dx$

$yx = -2\left|\int x\,dx + \int x\sin x\,dx\right|$

$yx = -2\left[\frac{x^2}{2} + (-x\cos x + \sin x)\right] + c$

$\Rightarrow yx = x^2 - 2x\cos x + 2\sin x + c$

At x = π, y = π + 2

$\Rightarrow (\pi + 2)\,\pi = \pi^2 + 2\pi + c$

$\Rightarrow \pi^2 + 2\pi = \pi^2 + 2\pi + c$

$\Rightarrow c = 0$

At $x = \frac{\pi}{2}$

$y.\frac{\pi}{2} = \left(\frac{\pi}{2}\right)^2 + 0 + 2 + 0 \Rightarrow y.\frac{\pi}{2} = \left(\frac{\pi}{2}\right)^2 + 2$

$y \quad \frac{\ }{2} \quad \frac{4}{\ }$

2. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2} [[x] - \sin x]\, dx$ is equal to :

(1) $-\pi$ (2) 1 (3) π (4) 0

Ans. (1)

Sol. $\int_{-\pi/2}^{\pi/2} [[x] - \sin x]\, dx$

$= \int_{-\pi/2}^{\pi/2} [[x] - \sin x]\, dx$

$= \int_{-\pi/2}^{\pi/2} ([-\sin x] + [x])\, dx \quad (\because [x + I] = [x] + I)$

Use property $\int_{-a}^{a} f(x)dx = \int_{0}^{a} (f(x) + f(-x))dx$

$= \int_{0}^{\pi/2} ([-\sin x] + [x] + [\sin x] + [-x])dx$

$= \int_{0}^{\pi/2} (-1 - 1)dx \quad \{\because [x] + [-x] = -1, x \notin I\}$

$= -2(x)\big|_{0}^{\pi/2}$

$= -\pi$

3. The lines x = ay – 1 = z and x = 3y – 2 = bz – 2, (ab ≠ 0) are coplanar if :

(1) a = 1, b ∈ R – {0} (2) a = 2, b = 3 (3) b = 1, a ∈ R – {0} (4) a = 2, b = 2

Ans. (3)

Sol. $\frac{x-0}{1} = \frac{y-\frac{1}{a}}{\frac{1}{a}} = \frac{z-2}{1}$ ....(1)

$\frac{x-0}{1} = \frac{y-\frac{2}{3}}{\frac{1}{3}} = \frac{z-\frac{2}{b}}{\frac{1}{b}}$ ....(2)

For coplanar :

$$\begin{vmatrix} a_2 - a_1 & b_2 - b_1 & c_2 - c_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & \frac{2}{3} \quad \frac{1}{a} \quad \frac{2}{b} & 2 \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} \quad 0$$

$$\frac{2}{3} \quad \frac{1}{a} \quad \frac{1}{b} \quad 1 \qquad \frac{2}{b} \quad 2 \quad \frac{1}{3} \quad \frac{1}{a} \qquad 0$$

$$\frac{1}{a} \quad \frac{2}{3} \quad \frac{1}{b} \quad 1 \qquad \frac{2}{b} \quad 2 \quad \frac{1}{3} \quad \frac{1}{a} \qquad 0$$

$\Rightarrow$ $(3 - 2a)(1 - b) + (2 - 2b)(a - 3) = 0$

$\Rightarrow 3 - 3b - 2a + 2ab + 2a - 6 - 2ab + 6b = 0$

$\Rightarrow 3b - 3 = 0$

$\Rightarrow b = 1, a \in R - \{0\}$

4. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(1) $\frac{\sqrt{5} \quad 1}{4}$ (2) $\frac{\sqrt{5} \quad 1}{4}$ (3) $\frac{\sqrt{5} \quad 1}{2}$ (4) $\frac{\sqrt{2} \quad 1}{2}$

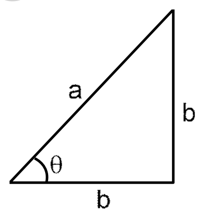
Ans. (3)

Sol. Let a > b > c

$\sin \quad \frac{c}{a}$

$\frac{1}{a} \quad \frac{1}{b} \quad \frac{1}{c}$

$\frac{1}{c^2} \quad \frac{1}{a^2} \quad \frac{1}{b^2}$



$1 \quad \frac{c^2}{a^2} \quad \frac{c^2}{b^2}$

$1 \quad \frac{c^2}{a^2} \quad \frac{c^2}{a^2 \quad c^2}$ [As $a^2 = b^2 + c^2$]

$1 \quad \sin^2 \quad \frac{1}{\frac{a^2}{c^2} \quad 1} \quad \sin^2 \quad \frac{1}{\operatorname{cosec}^2 \quad 1}$

$1 \quad \frac{1 \quad \sin^2 \quad 1}{\operatorname{cosec}^2 \quad 1} \quad \sin^2 \quad \operatorname{cosec}^2 \quad 3$

Let $\sin^2\theta = t$

$t \quad \frac{1}{t} \quad 3$

Solving above equation we get

$t = \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ (rejected)

So, $\sin\theta = \sqrt{\frac{3-\sqrt{5}}{2}} \Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

5. Let $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$, where $f(x) = \log_e\left(x + \sqrt{x^2+1}\right), x \in R$. Then which one of the following is correct ?

(1) $\sqrt{2}g(1) = g(0)$ (2) $g(1) + g(0) = 0$ (3) $g(1) = \sqrt{2}g(0)$ (4) $g(1) = g(0)$

Ans. (1)

Sol. $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx, f(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

Put t = 1 in g(t), we get

$g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4} + \log_e\left(x + \sqrt{x^2+1}\right)\right) dx$

Put t = 0 in g(t), we get

$g(0) = \int_{-\pi/2}^{\pi/2} \cos\left(\log_e\left(x + \sqrt{x^2+1}\right)\right)$

Since, $\sin\left(\log_e\left(x + \sqrt{x^2+1}\right)\right)$ is odd function

So $g(t) = \frac{1}{\sqrt{2}} \int_{-\pi/2}^{\pi/2} \cos\left(\log_e\left(x + \sqrt{x^2+1}\right)\right) dx$

$\sqrt{2}g(t) = g(0)$

6. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line y = x is :

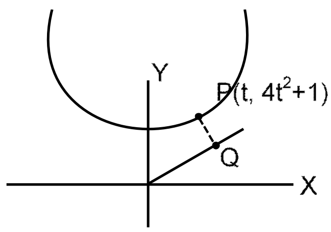
(1) $2(3x-y)^2 + (x-3y) + 2 = 0$ (2) $2(x-3y)^2 + (3x-y) + 2 = 0$

(3) $(3x-y)^2 + (x-3y) + 2 = 0$ (4) $(3x-y)^2 + 2(x-3y) + 2 = 0$

Ans. (1)

Sol. Let $P(t, 4t^2 + 1)$

Foot of perpendicular from P to y = x is



$Q\left(\frac{4t^2+t+1}{2}, \frac{4t^2+t+1}{2}\right)$

$\Rightarrow$ mid point of P and Q is

$$M \quad \frac{4t^2 \quad 3t \quad 1}{4}, \frac{12t^2 \quad t \quad 3}{4}$$

Locus of M is $2(3x - y)^2 + (x - 3y) + 2 = 0$

7. The value of $\tan \ 2\tan^{1} \frac{3}{5} \quad \sin^{1} \frac{5}{13}$ is equal to :

(1) $\frac{181}{69}$ (2) $\frac{220}{21}$ (3) $\frac{151}{63}$ (4) $\frac{291}{76}$

Ans. (2)

Sol. $\tan \ \tan^{1} \frac{\frac{6}{5}}{1 \quad \frac{9}{25}} \quad \tan^{1} \frac{5}{12}$

$$\tan \ \tan^{1} \frac{15}{8} \quad \tan^{1} \frac{5}{12} \quad \frac{\frac{15}{8} \quad \frac{5}{12}}{1 \quad \frac{15}{8} \ \frac{5}{12}} \quad \frac{220}{21}$$

8. If f : R $\rightarrow$ R is given by f(x) = x + 1, then the value of

$$\lim_{n} f(0) \quad f \frac{5}{n} \quad f \frac{10}{n} \quad ..... \quad f \frac{5(n \quad 1)}{n}$$

(1) $\frac{3}{2}$ (2) $\frac{7}{2}$ (3) $\frac{5}{2}$ 4) $\frac{1}{2}$

Ans. (2)

Sol. $\lim_{n} \frac{1}{n} \sum_{r \ 0}^{n \ 1} f \frac{5r}{n} \quad \int_0^1 f(5x)dx \quad \int_0^1 (5x \quad 1)dx$

$$\frac{5x^2}{2} \quad x \Big|_0^1 \quad \frac{5}{2} \quad 1 \quad \frac{7}{2}$$

9. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is:

(1) greater than $\frac{1}{2}$ (2) exactly equal to $\frac{1}{2}$

(3) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$ (4) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

Ans. (1)

Sol. $P(A) + P(B) - 2P(A \cap B) = 1 - K$

$P(A) + P(C) - 2P(A \cap B) = 1 - 2K$

$P(B) + P(C) - 2P(B \cap C) = 1 - K$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$$\frac{3 \quad 4k}{2} \quad k^2 \quad \frac{2k^2 \quad 4k \quad 3}{2}$$

$\because$ The value of $2k^2 - 4k + 3$ is greater than 1

$$P(A \quad B \quad C) \quad \frac{1}{2}$$

10. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (–4, 1) and having their centres on the circumference of the circle

(1) 7 (2) 11 (3) 5 (4) 3

Ans. (3)

Sol. $x^2 + y^2 + 2x + 4y - 4 = 0$

$(x + 1)^2 + (y + 2)^2 = 3^3$

So, $r \quad \sqrt{(3\cos \quad 3)^2 \quad 3(3\sin \quad 3)^2}$

$$3\sqrt{\cos^2 \quad 1 \quad 2\cos \quad \sin^2 \quad 1 \quad 2\sin}$$

$$3\sqrt{3 \quad 2(\cos \quad \sin\ )}$$

$$r_1 \quad 3\sqrt{3 \quad 2\sqrt{2}}$$

$$r_2 \quad 3\sqrt{3 \quad 2\sqrt{2}}$$

$$\frac{r_1}{r_2} \quad \frac{3\sqrt{3 \quad 2\sqrt{2}}}{3\sqrt{3 \quad 2\sqrt{2}}} \quad 3 \quad 2\sqrt{2}$$

On comparing with $\frac{r_1}{r_2} \quad a \quad b\sqrt{2}$

$a + b = 5$

11. Let $f : R \quad \overline{6} \quad R$ be defined by $f(x) \quad \frac{5x \quad 3}{6x}$. Then the value of a for which (f0f) (x) = x, for all $x \quad R \quad \overline{6}$, is :

(1) No such α exists (2) 6 (3) 8 (4) 5

Ans. (4)

Sol. $f(x) \quad \frac{5x \quad 3}{6x}$

$$f(f(x)) \quad \frac{5\ \frac{5x \quad 3}{6x} \quad 3}{6\ \frac{5x \quad 3}{6x}} \quad x$$

$$\frac{25x \quad 15 \quad 18x \quad 3}{30x \quad 18 \quad 6x \qquad {}^{2}} \quad x$$

$\Rightarrow 25x + 15 + 18x - 3\alpha = 30x^2 + 18x - 6\alpha x^2 + \alpha^2 x$

$\Rightarrow 25x - 15 - 3\alpha = 30x^2 - 6\alpha x^2 + \alpha^2 x$

$\Rightarrow 6(5 - \alpha)x^2 + (\alpha - 5)(\alpha + 5)x + 3(\alpha - 5) = 0$

$\Rightarrow \alpha = 5$

12. The sum of all the local minimum values of the twice differentiable function f : R $\rightarrow$ R defined by f(x)

$f(x) \quad x^3 \quad 3x^2 \quad \frac{3f''(2)}{2}x \quad f''(1)$ is :

(1) – 27 (2) – 22 (3) 5 (4) 0

Ans. (1)

Sol. $f'(x) \quad 3x^2 \quad 6x \quad \frac{3}{2}f''(2)$

$f''(x) = 6x - 6$

$f''(1) = 0$ & $f''(2) = 6$

Then the local minimum value $f''(x) = 0$

$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$

$X = -1$ and $x = 3$

Local minimum at x = 3

So local minimum value y(3) = f(3)

$$3^3 \quad 3^3 \quad 3 \quad \frac{6}{2} \quad 3 \quad 0 \qquad 27$$

13. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of |a – b| is equal to :

(1) 9 (2) 7 (3) 11 (4) 1

Ans. (4)

Sol. Mean = 10

$$\frac{7 \quad 10 \quad 11 \quad 15 \quad a \quad b}{6} \quad 10$$

a + b = 17 …..(1)

variance $\frac{20}{3}$

$$\frac{49 \quad 100 \quad 121 \quad 225 \quad a^2 \quad b^2}{6} \quad 100 \quad \frac{20}{3}$$

$a^2 + b^2 = 145$

$(a + b)^2 = 289$

$(a-b)^2 = (a+b)^2 - 4ab$

$(a-b)^2 = 289 - 288 = 1$

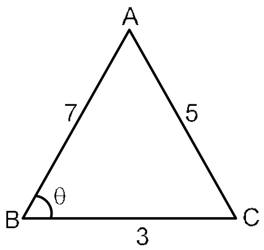
$|a-b| = 1$

14. In a triangle ABC, if $|\overrightarrow{BC}| = 3, |\overrightarrow{CA}| = 5$ and $|\overrightarrow{BA}| = 7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :

(1) $\frac{11}{2}$ (2) $\frac{13}{2}$ (3) $\frac{15}{2}$ (4) $\frac{19}{2}$

Ans. (1)

Sol.



Projection of $\overrightarrow{BA}$ on $\overrightarrow{BC} = 7\cos\theta$

$4\ \frac{7^2 + 3^2 - 5^2}{2 \cdot 7 \cdot 3} = \frac{11}{2}$

15. If the real part of the complex number $(1-\cos\theta + i2\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\,dx$ is equal to :

(1) 0 (2) 1 (3) –1 (4) 2

Ans. (2)

Sol. $z = (1-\cos\theta + 2i\sin\theta)^{-1}$

$z = \frac{1}{1-\cos\theta + i2\sin\theta} \times \frac{(1-\cos\theta - i2\sin\theta)}{(1-\cos\theta - i2\sin\theta)}$

$z = \frac{1-\cos\theta - i2\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$

$\text{Re}(z) = \frac{1-\cos\theta}{(1-\cos\theta)^2 + 4\sin^2\theta} = \frac{1}{5}$

$\theta = 0, \frac{\pi}{2}$

But, $\theta \in (0, \pi)$

So, $\theta = \frac{\pi}{2}$

Now, $\int_0^{\pi/2} \sin x\,dx = 1$

16. The value of $k \in R$, for which the following system of linear equations

$3x - y + 4z = 3,$

$x + 2y - 3z = -2,$

$6x + 5y + kz = -3,$

has infinitely many solutions, is :

(1) 3 (2) – 5 (3) 5 (4) – 3

Ans. (2)

Sol.
$$\begin{vmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \\ 6 & 5 & k \end{vmatrix}$$

$\Delta = 3(2k + 15) + 1(k + 18) + 4(5 - 12) = 0$

$\Rightarrow \quad 7k + 35 = 0$

$\Rightarrow \quad k = -5$

17. If sum of the first 21 terms of th series $\log_9 \frac{1}{2}x \quad \log_9 \frac{1}{3} \quad \log_9 \frac{1}{4}x \quad ....$, where $x > 0$ is 504, then x is equal to :

(1) 243 (2) 7 (3) 81 (4) 9

Ans. (3)

Sol. $2 \log_9 x + 3 \log_9 x + 4\log_9 x$ ..... 21 terms

$$(2 \quad 3 \quad 4 \quad 5 \quad ....22)\log_9 k \quad \frac{21}{2}(2 \quad 22)\log_9 x$$

$= 21 \times 12 \log_9 x$

$= 252 \log_9 x = 504$

$\text{Log}_9 x = 2 \Rightarrow x = 81$

18. For the natural numbers m, n, if $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + ... + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m + n)$ is equal to :

(1) 100 (2) 64 (3) 88 (4) 80

Ans. (4)

Sol. $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + ...$

a_1 is coefficient of $y = 1.\ {}^nC_1 - {}^mC_1.\ 1 = 10$

$\Rightarrow n - m = 10 \qquad ....(1)$

a_2 is coefficient of $y^2 = {}^mC_2 + {}^nC_2 - {}^mC_1\ {}^nC_1 = 10$

$\Rightarrow m(m - 1) + n(n - 1) - 2mn = 20$

$m^2 + n^2 - 2mn - (m + n) = 20$

$(m - n)^2 - (m + n) = 20$

$(m + n) = 80$

19. Consider the line L given by the equation $\frac{x\quad 3}{2}\quad\frac{y\quad 1}{1}\quad\frac{z\quad 2}{1}$. Let Q be the mirror image of the point (2, 3, –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?

(1) (1, 1, 1) (2) (–1, 1, 2) (3) (1, 2, 2) (4) (1, 1, 2)

Ans. (3)

Sol. Let A (2, 3, –1)

Let image of A(2, 3, –1) in the line mirror

$\frac{x\quad 3}{2}\quad\frac{y\quad 1}{1}\quad\frac{z\quad 2}{1}$ is Q(α, β, γ)

$\frac{\quad 2}{2},\frac{\quad 3}{2},\frac{\quad 1}{2}$ lines on $\frac{x\quad 3}{2}\quad\frac{y\quad 1}{1}\quad\frac{z\quad 2}{1}$

$\frac{\quad 4}{4}\quad\frac{\quad 1}{2}\quad\frac{\quad 5}{2}$ ....(1)

Also AQ $\perp$ to given line (L)

$\Rightarrow 2(\alpha - 2) + (\beta - 3) + (\gamma + 1) = 0$

$\Rightarrow 2\alpha + \beta + \gamma - 6 = 0$ ....(2)

by solving (1) and (2)

we get $\alpha = 2$, $\beta = -2$, $\gamma = 4$ $\Rightarrow$ Q(2, –2, 4)

Now equation of plane P which passes through Q(1, –1, 5) and perpendicular to the line L is

$2(x - 2) + 1\,(y + 2) + 1\,(z - 4) = 0$

$2x + y + z = 6$

Hence point (1,2,2) lies in it

20. Consider the following three statements :

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then which of the following statements is correct ?

(1) (A) and (C) are true while (B) is false

(2) (A) and (B) are false while (C) is true

(3) (A) is false, but (B) and (C) is true

(4) (A) is true while (B) and (C) are false

Ans. (1)

Sol. Truth table $p \rightarrow q$

p	q	p q
T	T	T
T	F	F
F	T	T
F	F	T

A is true, B is false, C is true

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Let a curve y = y(x) be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}\,dy$$

If it intersects y-axis at y = –1, and the intersection point of the curve with x-axis is (α, 0), then e^{α} is equal to ________.

Ans. (2)

Sol. $$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}\,dy$$

$$\int \frac{\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx}{\sqrt{e^{2x}-1}} = \int dy$$

Put $\cos^{-1}(e^{-x}) = t$

$$\frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx = dt$$

$$\frac{dx}{\sqrt{e^{2x}-1}} = dt$$

$$\int \cos\frac{t}{2}\,dt = y + c$$

$$2\sin\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) = y + c$$

At x = 0 $\Rightarrow$ y = – 1

c = 1

$$y = 2\sin\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) - 1$$

y = 0 then x = α

$$2\sin\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) = 1$$

$$e^{-x} = \frac{1}{2} \Rightarrow x = \ln 2$$

So, the value of $e^{\alpha} = 2$

2. Let A = $\{a_{ij}\}$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then det (3 Adj(2 A^{-1})) is equal to ________.

Ans. (108)

Sol. A $\begin{matrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix}$

So, $|A| \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$

$= 2(4 - 1) + 1(-2 + 1) + 1(1 - 2)$

$= 2(3) + 1(-1) + 1(-1)$

$= 4$

$|3Adj(2A^{-1})| = 3^3\,|Adj(2A^{-1})| = 3^3 \times |2A^{-1}|^2$

$3^3 \quad 2^6 \quad |A^{1}|^2 \quad 3^3 \quad 2^6 \quad \frac{1}{|A|^2} \quad 108$

3. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0$, $x > 0$, is _____.

Ans. (1)

Sol. $\log_{(x+1)}((2x + 5)(x + 1)) + \log_{(2x+5)}(x+1)^2 = 4$

$1 + \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) = 4$

Put $\log_{(x+1)}(2x + 5) = t$

$1 \quad t \quad \frac{2}{t} \quad 4$

$t^2 + t + 2 = 4t \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, t = 2$

For t = 1	For t = 2
$2x + 5 = x + 1$	$2x + 5 = (x + 1)^2$
$\Rightarrow x = -4$ (rejected)	$x = 2, x = -2$ (rejected)

4. For p > 0, a vector $\vec{V}_2 \quad 2\hat{i} \quad (p \quad 1)\hat{j}$ is obtained by rotating the vector $\vec{V}_1 \quad \sqrt{3}p\hat{i} \quad \hat{j}$ by an angle θ about origin in counter clockwise direction. If tan $\frac{(\quad\sqrt{3} \quad 2)}{(4\sqrt{3} \quad 3)}$, then the value of α is equal to _________ .

Ans. (6)

Sol. $|v_1| = |v_2|$

$\Rightarrow 3p^2 + 1 = 4 + (P+1)^2$

$\Rightarrow 2p^2 - 2p - 4 = 0$

$\Rightarrow p^2 - p - 2 = 0$

$\Rightarrow (p - 2)(p + 1) = 0$

$\Rightarrow p = -1, 2$

$\Rightarrow$ p = 2 (p > 0)

If angle b/w $\vec{v}_1$ & $\vec{v}_2$ is θ

$$\cos \quad \frac{\vec{v}_1 \quad \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} \quad \frac{2\sqrt{3}\hat{i} \quad \hat{j} \quad 2\hat{i} \quad 3\hat{j}}{\sqrt{13}\sqrt{13}}$$

$$\cos \quad \frac{4\sqrt{3} \quad 3}{13}$$

$$\tan \quad \frac{\sqrt{112 \quad 24\sqrt{3}}}{4\sqrt{3} \quad 3}$$

$$\tan \quad \frac{2\sqrt{28 \quad 6\sqrt{3}}}{4\sqrt{3} \quad 3} \quad \frac{2 \;\; 3\sqrt{3} \quad 1}{4\sqrt{3} \quad 3}$$

$$\frac{6\sqrt{3} \quad 2}{4\sqrt{3} \quad 3} \qquad 6$$

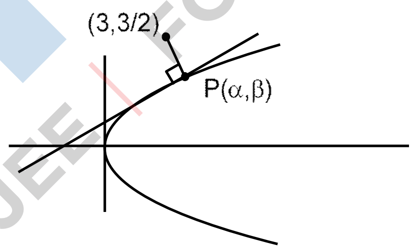
5. If the point on the curve $y^2 = 6x$, nearest to the point $3, \frac{3}{2}$ is (α, β) is equal to ________ .

Ans. (9)

Sol. $y^2 = 6x$

$2yy' = 6$

$$\frac{dy}{dx} \quad \frac{3}{}$$



$$\frac{}{3} \qquad \frac{3/2}{3}$$

$$\frac{}{3} \quad \frac{2 \quad 3}{2 \quad 6}$$

$-\beta(2\alpha - 6) = 6\beta - 9$

$6\beta - 2\alpha\beta = 2\beta - 9$

$$\frac{9}{2} \qquad \frac{9}{2}$$

$\therefore \; \beta^2 = 6\alpha$

$$\frac{81}{4 \;\; ^2} \quad 6$$

$$^2 \quad \frac{27}{8} \quad \frac{3}{2}, \; ^2 \quad 9 \qquad 3$$

$$\frac{3}{2}, \quad 3$$

$2(\alpha + \beta) = 9$

6. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to ________.

Ans. (7)

Sol. Let $S = \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = \sum_{n=1}^{\infty} \frac{a_n}{8^n} = \frac{a_1}{8^1} + \frac{a_2}{8^2} + \frac{a_3}{8^3} + ...$ ....(1)

And $a_{n+2} = 2a_{n+1} = a_n$

$$S = \sum_{n=1}^{\infty} \frac{a_{n+2} - 2a_{n+1}}{8^n}$$

$$= \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^n} - \frac{2a_{n+1}}{8^n}$$

$$= \sum_{n=1}^{\infty} \frac{8^2 a_{n+2}}{8^{n+2}} - \frac{2 \cdot 8 \cdot a_{n+1}}{8^{n+1}}$$

$$S = 64 \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} - 16 \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}}$$

$$S = 64\left(\frac{a_3}{8^3} + \frac{a_4}{8^4} + ...\right) - 16\left(\frac{a_2}{8^2} + \frac{a_3}{8^3} + ...\right)$$

$$S = 64\left(S - \frac{a_1}{8} - \frac{a_2}{64}\right) - 16\left(S - \frac{a_1}{8}\right), \text{ from (i)}$$

$\because a_1 = a_2 = 1$

$\Rightarrow (47)\, s = 7$

7. For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)......(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$, where $\alpha > 0$. Then the value of $100\left(\frac{A_{14} + A_{15}}{A_{13}}\right)^2$ is equal to ________ .

Ans. (9)

Sol. $\frac{1}{\alpha(\alpha+1)(\alpha+2)......(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$

$$\frac{1}{\alpha(\alpha+1)(\alpha+2)......(\alpha+20)} = \frac{A_0}{\alpha} + \frac{A_1}{\alpha+1} + \frac{A_2}{\alpha+2} + \frac{A_3}{\alpha+3} + ... + \frac{A_{20}}{\alpha+20}$$

$\Rightarrow 1 = A_0 (\alpha + 1)(\alpha + 2) .... (\alpha + 20) + A_1 (\alpha)(\alpha + 2)(\alpha + 3) .... (\alpha + 20)$

$+ ...... + A_{20}\alpha (\alpha + 1)(\alpha + 2)(\alpha + 2) ...... (\alpha + 19)$

$\Rightarrow$ Put $\alpha = -14 \Rightarrow$ $A_{14} = \frac{1}{(-14)(-13)(-12)......(-1)(1)(2)......(6)}$

$$A_{15} = \frac{1}{(-15)(-14)(-13)......(-1)(1)(2)......(5)}$$

$$A_{13} \quad \frac{1}{(\ \ 13)(\ \ 12)......(\ \ 1)(1)(2)......(7)}$$

$$\frac{A_{15} \quad A_{14}}{A_{13}}^{2} \qquad \frac{A_{15}}{A_{13}} \quad \frac{A_{14}}{A_{13}}^{2}$$

$$\frac{6.7}{15.14} \quad \frac{7}{14}$$

$$\frac{1}{5} \quad \frac{1}{2}^{2} \qquad \frac{9}{100}$$

$$100 \ \frac{A_{15} \quad A_{14}}{A_{13}} \qquad 9$$

8. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circumcentre of triangle ABC, bisects line BC, and intersects y-axis at point $0, \frac{}{2}$, then the value of real number α is _________ .

Ans. (9)

Sol. Mid point of BC is $2, \frac{17}{2}$ and slope of BC is $\frac{1}{2}$.

Since a line passes through circumcentre of ΔABC and bisects the side BC is perpendicular bisector of side BC.

Equation of required line is

$$y \quad \frac{17}{2} \quad 2(x \quad 2)$$

$$2x \quad y \quad 4 \quad \frac{17}{2}$$

$$4x + 2y + 9 = 0$$

It intersects the y-axis at $0, \frac{}{2}$

$\Rightarrow -\alpha + 9 = 0$

$\Rightarrow \alpha =$

9. Let a function g : [0, 4] $\rightarrow$ R be defined as g(x) $\begin{matrix} \max\{t^3 \quad 6t^2 \quad 9t \quad 3\}, \ 0 \quad x \quad 3 \\ 0 \quad t \quad x \\ 4 \quad x, \quad 3 \quad x \quad 4 \end{matrix}$ then the number of points in the interval (0, 4) where g(x) is NOT differentiable, is ________ .

Ans. (1)

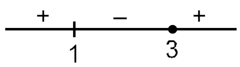
Sol. $f(t) = t^3 - 6t^2 + 9t - 3$

$f'(t) = 3t^2 - 12t + 9 = 0$

$= t^2 - 4t + 3 = 0$

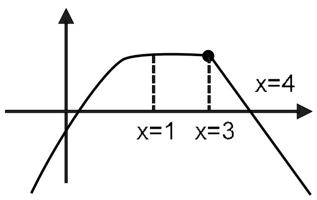
$t = 1, 3$

$f(1) = 1$



$f(3) = -3$

$$g(x) \quad \begin{matrix} x^3 \quad 6x^2 \quad 9x \quad 3 & 0 \quad x \quad 1 \\ 1 & 1 \quad x \quad 3 \\ 4 \quad x, & 3 \quad x \quad 4 \end{matrix}$$



Function is non differentiable at x = 3

10. If $\lim\limits_{x \quad 0} \dfrac{xe^{x} \quad \log_e(1 \quad x) \quad x^2 e^{\ \ x}}{x\sin^2 x} \quad 10, \ , \ , \ , \ R$ then value of $\alpha + \beta + \gamma$ is ________.

Ans. (3)

Sol. $\lim\limits_{x \quad 0} \dfrac{x \ \ 1 \ \ x \ \ \dfrac{x^2}{2!} \ \ \dfrac{x^3}{3!} \ ..... \qquad x \ \ \dfrac{x^2}{2} \ \ \dfrac{x^3}{3} \ ..... \qquad x^2 \ \ 1 \ \ x \ \ \dfrac{x^2}{2!} \ \ \dfrac{x^3}{3!} \ .....}{x^3} \quad 10$

$\Rightarrow \alpha - \beta = 0, \Rightarrow \alpha = \beta$

$\overline{2} \qquad 0 \qquad \dfrac{3}{2}$

$\overline{2} \ \ \overline{3} \qquad 10 \qquad \overline{2} \ \ \overline{3} \ \ \dfrac{3}{2} \quad 10 \qquad \dfrac{3 \quad 2 \quad 9}{6} \quad 10$

$\because \ \beta = 6, \alpha = 6, \gamma = -9$

So, the value of $\alpha + \beta + \gamma = 3$